

Neo-Gregorian Calendar and Septenary Neo-Gregorian Calendar

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ABSTRACT: The Gregorian calendar used in daily life needs to represent not only the date but also the day of the week. Its date and the day of the week have no relationship and are independent. This paper develops a New Calendar, called the Neo-Gregorian Calendar. The date of the Neo-Gregorian Calendar contains the day of the week, so that the date and the day of the week are no longer independent. The day of the week can be immediately known through the date, so that the day of the week can be omitted, which will bring great convenience in work and study. This Neo-Gregorian Calendar is neat, symmetrical, concise and clear, and beautifies the traditional Gregorian calendar.

Keywords: calendar, Gregorian calendar, Neo-Gregorian calendar, matrix Gregorian calendar

If the first day of a certain month in the Gregorian calendar is Wednesday, the first day of this month will be changed to the 3 day, the second day of this month will be changed to the 4 day,... Generally, the k -th day of this month will be changed to the $k+2$ day. Similarly, if the first day of a month in the Gregorian calendar is Friday, the first day of the month will be changed to the 5 day, the second day of the month will be changed to the 6 day,... Generally, the k -th day of this month will be changed to the $k+4$ day. In this way, a Neo-Gregorian Calendar is obtained. The date of the Neo-Gregorian Calendar relative to the date of the Gregorian calendar is called the new Gregorian date, abbreviated as the new date. According to the new date of a day in this Neo-Gregorian calendar, we can immediately know the day of the week of that day, because it is the remainder of the new date of that day divided by 7. In addition to February in the Gregorian calendar, as long as you know the new date or the day of the week of the first day of a month in the Gregorian calendar, you will know the new date of any day of the month. As long as you know the new date or the day of the week of the last day of a month in the Gregorian calendar, you will know the new date of the first day of the month, so you will know the new date and the day of the week of any day of the month! Generally, as long as you know the new date or the day of the week of Day m of a month in the Gregorian calendar, you will know the new date of the first day of the month, so you will know the new date and the day of the week of any day of the month. If the new date of Day k of a certain month in the Gregorian calendar is n , then $k \leq n \leq k+6$. For example, if the first day of October in the Gregorian calendar is Wednesday, the new date of the first day of the month is 3, the new date of the second day is 4, and the new date of Day k is $k+2$ day. If the new date of the last day of June in the Gregorian calendar is 35 day, because June in the Gregorian calendar has 30 days, and because $35=30+5$, the first day of the Gregorian calendar in this month is Saturday, so the new date of the first day of this month is 6 day, and the new date of Day k of this month is $k+5$ day, another method is that the remainder of 35 divided by 7 is 0, June 30 is Sunday, $30-28=2$, June 2nd is Sunday, and June the first is Saturday, so the new date of Day k of this month is $k+5$ day. and the remainder of $k+5$ divided by 7 is its the day of the week. If the last day of August in the Gregorian calendar is Friday, that is, the 31st day of August is Friday, $31-28=3$, that is, the 3rd day of August is Friday, so the 2nd day of August is Thursday, the 1st day of August is Wednesday, the new date of the 1st day of August is the 3 day, and the new date of the k -th day of August is the $k+2$ day. If the new date of the 12th day of a month in the Gregorian calendar is the 15 day, it is obvious that the 1st day of the month is Thursday ($15-12+1$), and the new date of the k -th day of the month is the $k+3$ day, another method is that the remainder of 15 divided by 7 is 1, the 12th day of the Gregorian calendar in the month is Monday, $12-7=5$, the 5th day is Monday, the 4th day is Sunday, the 3rd day is Saturday, the 2nd day is Friday, the 1st is Thursday, the new date of the k -th day of the month is the $k+3$ day. If the 19th day of a month in the Gregorian calendar is Tuesday, because $19-14=5$, so, the 5th day of this month is Tuesday, the 4th day is Monday, the 3rd day is Sunday, the 2nd day is Saturday, and the 1st day is Friday, the new date of the 1st day of

this month is 5 day, the new date of Day k of this month is $k+4$ day, and the remainder of $k+4$ divided by 7 is its the day of the week.

The biggest advantage of this Neo-Gregorian calendar is that as long as you know the new date of a day, you can know the the day of the week of that day, so the day of the week can be omitted. Obviously, it is not more difficult than the Gregorian calendar in daily life! Note: the Neo-Gregorian calendar may or may not involve a conversion relationship between the new date of a certain day and its Gregorian calendar date. When such a relationship does exist, the conversion between them is obviously very simple, as described above! This conversion relationship is essentially the mutual transformation between the Gregorian calendar date of that day and the new date of that day.

The Gregorian calendar involves weeks, and each week has seven days, so it would be best to represent Gregorian dates in base 7. However, expressing ordinary Gregorian dates in base 7 is not particularly meaningful. But when it comes to New Gregorian dates—that is, the Neo-Gregorian Calendar dates—representing them in base 7 is quite interesting. Besides retaining all the functions of the decimal New Gregorian dates, it has a significant advantage: the units digit of a base-7 New Date directly indicates the day of the week! This is especially meaningful, as it eliminates the need to calculate the day of the week. As long as you know the base-7 representation of a New Date, you naturally know the corresponding day of the week—it's incredibly convenient. Let's continue studying the Neo-Gregorian Calendar using base 7

As long as you know the base-7 New Date or the day of the week for a certain day of a month in the Gregorian calendar, you can determine the base-7 New Date for the first day of that month, and from there, you can find the base-7 New Date and the day of the week for any day of that month. All calculations must be carried out in base 7. Give examples to illustrate:

If the New Date for the last day of June in the Gregorian calendar is 51 (in base 7), and since June has 42 days in base 7 (30 in decimal is 42 in base 7), and because $51 = 42 + 6$, the first day of this month is a Sunday. Therefore, the New Date for the first day of this month is 10 (7 in decimal is 10 in base 7). The New Date for the k -th day of this month is $k+6$. Taking the units digit of $k+6$ gives the day of the week for that day. Another method: the New Date for that day is 51, so that day is a Monday (since the units digit of 51 is 1). Calculating $42 - 40 = 2$ (40 in base 7 is 28 in decimal), the second day of this month is Monday, so the first day of this month is Sunday. The New Date for the k -th day of this month is $k+6$. Taking the units digit of $k+6$ gives the day of the week for that day.

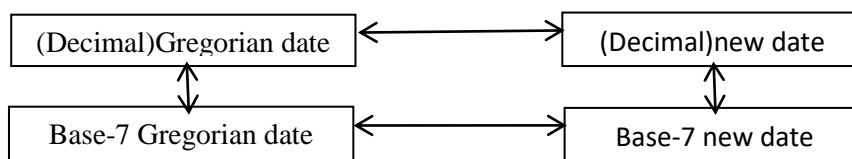
If the last day of August in the Gregorian calendar is a Friday, meaning the 43rd day (31 in decimal is 43 in base 7) of August is a Friday, then $43 - 40 = 3$, so the 3rd day of August is a Friday. Therefore, the 2nd day of August is a Thursday, and the 1st day of August is a Wednesday. The New Date for the first day of August is 3. The New Date for the k -th day of August is $k+2$. Taking the units digit of $k+2$ gives the day of the week for that day.

If the New Date for the 12th day (in base 7) of a certain Gregorian month is 15 (in base 7), then it is clear that the first day of this month is a Thursday (since $15 - 12 + 1$). The New Date for the k -th day of this month is $k+3$. Taking the units digit of $k+3$ gives the day of the week for this day. Another method: the New Date for the 12th day (in base 7) is 15 (in base 7), so that day is a Friday. Since $12 - 10 = 2$, the 2nd day of this month is Friday, so the 1st day is Thursday. The New Date for the k -th day of this month is $k+3$. Taking the units digit of $k+3$ gives the day of the week for that day.

If the 16th day (in base 7) of a certain Gregorian month is a Wednesday, then since $16 - 10 = 6$, the 6th day of this month is Wednesday. Thus, the 5th day is Tuesday, the 4th day is Monday, the 3rd day is Sunday, the 2nd day is Saturday, and the 1st day is Friday. The New Date for the first day of this month is 5, The New Date for the k -th day of this month is $k+4$. Taking the units digit of $k+4$ gives the day of the week for that day.

Just like the decimal Neo-Gregorian Calendar, the base-7 Neo-Gregorian Calendar may or may not have a conversion relationship between the base-7 New Date of a given day and its base-7 Gregorian date. When such a relationship does exist, the conversion between them is as described above and is clearly very simple! This conversion relationship essentially involves the interchange between the base-7 Gregorian date and the base-7 New Date for that day. As long as you are familiar with base-7, it is just as simple as the earlier decimal case—in fact, even simpler! Note that here, everything must be in base 7. The Gregorian date cannot be in decimal while the conversion uses base-7 operations, and vice versa. The same requirement applies to the decimal Neo-Gregorian Calendar: everything must be in decimal!

Each day in the Gregorian calendar has four dates: the (decimal) Gregorian date, the (decimal) New Date, the base-7 Gregorian date, and the base-7 New Date. The following flowchart clearly illustrates how to convert between these four dates!



Among them, the horizontal conversion refers to the interchange between Gregorian dates and New Dates, which has been discussed earlier; the vertical conversion refers to the interchange between decimal and base-7 representations of numbers. Thus, both are extremely simple!

From the flowchart above, it can be seen that the conversion between the decimal Gregorian date and the base-7 New Date for a given day of a month must be carried out in two steps, and there are two methods: ① First, convert the decimal Gregorian date of that day into a decimal New Date, and then convert that decimal New Date into a base-7 New Date, and vice versa; ② First, convert the decimal Gregorian date of that day into a base-7 Gregorian date, and then convert that base-7 Gregorian date into a base-7 New Date, and vice versa.

Similarly, the conversion between the decimal New Date and the base-7 Gregorian date for a given day of a month must also be carried out in two steps, and there are likewise two methods. For example: Suppose the decimal New Date for a certain day in a Gregorian month is 26, and 26 in base-7 is 35, then the base-7 New Date for that day is 35.

Assume that the 1st day of a certain Gregorian month is a Wednesday. Find the base-7 New Date for the 21st day of that month in decimal Gregorian. The decimal New Date for the 21st is $21 + 2 = 23$, and 23 in base-7 is 32, so the base-7 New Date for that day is 32. Another method: 21 in decimal is 30 in base-7, and the base-7 New Date for 30 is $30 + 2 = 32$, which is the required result.

Assume that the 1st day of a certain Gregorian month is a Saturday. Find the base-7 Gregorian date for the day whose decimal New Date is 15. The decimal Gregorian date for 15 is 10, and 10 in decimal is 13 in base-7, so 13 is the required result. Another method: the base-7 New Date for decimal New Date 15 is 21, and the base-7 Gregorian date for 21 is $21 - 5 = 13$, which is the required result.

Note that the first day of each Gregorian month can fall on any of seven possible days of the week: Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. In terms of numerical representation, for the numbers 1 through 6, they remain the same in both decimal and base-7 systems. Only the number 7 changes: in base-7, it is represented as 10, while in decimal, it remains 7. In summary, when performing conversions between different systems, the first day of each month plays a crucial role, yet its treatment is quite straightforward.

Below, a slightly improved version of the Gregorian calendar is presented.

January 2026

4	5	6	7	1	2	3
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	Table 1			

February 2026

7	1	2	3	4	5	6
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

Table 2

Table 1 and table 2 show the slightly improved Gregorian calendar in January and February 2026, which can be called the matrix Gregorian calendar.

The matrix Gregorian calendar of a month in the Gregorian calendar is constructed as follows:

1. week arrangement: the week is arranged from the day of the week corresponding to the first day of the Gregorian calendar of each month, increasing by one day every day (module 7) until the seventh day. The week is arranged in the top row, which is dynamically marked according to the starting day of each month to adapt to different months.

2. date arrangement: the date is arranged from the 1st of the Gregorian calendar of this month, and the line is changed every 7 days until the end of this month. It doesn't matter that the last line may be less than 7 days. The day of the week and date are aligned vertically and horizontally.

3. month and year display: the month and year are clearly marked on the very top.

4. example: Table 1 above shows the matrix Gregorian calendar for January 2026 of the Gregorian calendar. It is neat and orderly. Each line is fixed to 7 days (except the last line). Its week is 4567123 on the top line. Because the first day of the Gregorian calendar is Thursday, the week is arranged from 4, followed by 567123 (Friday on the 2nd, Saturday on the 3rd, and so on to Wednesday on the 7th). Table 2 shows the matrix Gregorian calendar of February 2026 in the Gregorian calendar. The first day of the Gregorian calendar is Sunday, the second day is Monday, and so on to the seventh day is Saturday. The week arrangement starts from seven, followed by 123456, and its week arrangement of 7123456 is also on the top row. It is not difficult to get the matrix Gregorian calendar of March, April and any month in the Gregorian calendar and their week arrangement. Obviously, in the matrix Gregorian calendar of January 2026 in the Gregorian calendar, you can quickly find the 27th and immediately make it clear that it falls on Tuesday. In contrast, finding the 27th and its day of the week in the traditional Gregorian calendar is not so fast and direct. In addition, this date arrangement (matrix Gregorian calendar) is simpler and more orderly than the traditional Gregorian calendar.

As long as you know the new date of a day, you will know the the day of the week for that day. Therefore, replace the Gregorian calendar date in Table 1 with the decimal new date and base-7 new date, and delete the top line to get:

04	05	06	07	08	09	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34				

Table 3

04	05	06	10	11	12	13
14	15	16	20	21	22	23
24	25	26	30	31	32	33
34	35	36	40	41	42	43
44	45	46				

Table 4

Table 3 is called the new matrix Gregorian calendar of January 2026 of the Gregorian calendar, and table 4 is called the base-7 new matrix Gregorian calendar of January 2026 of the Gregorian calendar. They do not need to be marked with day of the week! Obviously, they are simpler, more symmetrical and more beautiful than table 1! Table 4 is neat. The units digits of each column are the same, which is more orderly than table 3. The units digits of each date in Table 4 are the the day of the week for the day, but the remainder of the date in Table 3 divided by 7 is the the day of the week for the day. Therefore, table 4 is not only more orderly than table 3, but also easier to use! In other words, it is better to represent the Neo-Gregorian Calendar in base-7 than in decimal!

In the Neo-Gregorian Calendar, especially in the base-7 Neo-Gregorian Calendar, as long as you know the New Date, you immediately know the day of the week. Conversely, from the day of the week, you can infer the New Date. For example, if a certain day is a Wednesday, its base-7 New Date can only be one of five possibilities: 03, 13, 23, 33, or 43. These five New Dates are each exactly 7 days apart. Based on this relatively long time interval, one can determine which specific day it actually is.

At first glance, this system may sometimes require remembering the day of the week for the 1st of the month, which might seem troublesome. However, it is actually quite simple! This is because you only need to remember one number among 1, 2, 3, 4, 5, 6, or 10. Surely, remembering such a simple number over the course of a month is manageable. It is even simpler than asking, "What day was yesterday?" and answering, "It was Tuesday." In this sense, it is trivial and can be memorized effortlessly without conscious effort.

A date is merely a marker or identifier; there is no inherent need for it to start from 1. This is because a Gregorian date essentially indicates the nth day of a month. Calculating the nth day of a month or the number of days between two specific dates can clearly be done directly using the New Date system. In other words, whether the first day of a month starts from 1, or from 2, 3, etc., it makes no difference when calculating the nth day of that month (Gregorian date - 1 + 1, New Date - 2 + 1, etc.) or the number of days between two dates. Therefore, the starting point for the dates in a month is irrelevant, and the Gregorian calendar holds no inherent advantage over the Neo-Gregorian Calendar in this regard.

Consequently, for holidays, one only needs to remember that they fall on a specific nth day of a particular month. If one is familiar with base-7, it is even better to represent this nth day in base-7. For example:

- National Day is the 1st day of October.
- Labor Day is the 1st day of May.

For holidays like these, representing them with base-7 New Dates is superior to using Gregorian dates, even if it might seem superficially different. Why? Because once you know the base-7 New Date for the first day of a month, you instantly know the base-7 New Date for any day of that month, and thus immediately know the day of the week for any given day. In contrast, with the Gregorian calendar, knowing it is the first day of the month tells you nothing else.

For instance, suppose the first day of September corresponds to the New Date 10. Then, the 13th day of September (Teacher's Day) has a New Date of $13 + 10 - 1 = 22$. Clearly, this day is a Tuesday. Similarly, Women's Day falls on the 8th day of March, and its New Date can be easily calculated.

The most typical example is a birthday. One only needs to remember that their birthday is on the n th day of a certain month. Then, by adding the New Date of the first day of that month and subtracting 1, you obtain the New Date of the birthday. Conversely, given the New Date of a birthday, the birthday corresponds to the n th day of that month, where $n = (\text{New Date of birthday}) - (\text{New Date of the first day}) + 1$.

With the base-7 Neo-Gregorian Calendar, some holidays with variable dates actually become fixed. For example:

- Mother's Day corresponds to the base-7 New Date 20 in May.
- Father's Day corresponds to the base-7 New Date 30 in June.
- Thanksgiving corresponds to the base-7 New Date 34 in November.

From this, one can immediately determine that Father's Day and Mother's Day always fall on a Sunday, and Thanksgiving always falls on a Thursday. Clearly, these are things the Gregorian calendar cannot achieve.

Thus, the base-7 New Date system not only allows holidays with fixed Gregorian dates (like Teacher's Day) to be easily determined, but it also fixes holidays with variable Gregorian dates (like Mother's Day).

The significance of the base-7 New Date September 22 being Teacher's Day is as follows:

- ① This day is a Tuesday and corresponds to the 13th day (10 in base-7) of September.
- ② The New Date for the first day of September is $22 - 13 + 1 = 10$, meaning the first day of September is a Sunday.
- ③ The New Date for the last day of September (the 30th day in decimal, which is 42 in base-7) is $42 + 6 = 51$, so the last day of September is a Monday.
- ④ For any given day k in September (with k in base-7), its New Date is $k + 6$, and the units digit of $k + 6$ indicates its day of the week.

From this, it can be seen that while using the New Date to represent holidays may feel unfamiliar at first, it is highly valuable and rich in meaning. It offers far more than simply representing a holiday with a Gregorian date, which is just a date and nothing more.

In base-7, the dates of a month can certainly be represented by the first 52 (37 in decimal) natural numbers. Addition and subtraction in base-7 for numbers from 1 to 52 are extremely simple — even simpler than in decimal! Converting these 52 numbers between base-7 and decimal is also very straightforward. For example, base-7 26 in decimal is $2 \times 7 + 6 = 20$; base-7 34 in decimal is $3 \times 7 + 4 = 25$. That is, to convert a base-7 number to decimal, multiply its tens digit by 7 and add its units digit. Conversely, decimal 19 in base-7 is 25, obtained by dividing 19 by 7: quotient 2, remainder 5. Decimal 28 in base-7 is 40, obtained by dividing 28 by 7: quotient 4, remainder 0. That is, to convert a decimal number to base-7, divide it by 7; the quotient becomes the tens digit, and the remainder becomes the units digit.

In summary, for recording and representing dates, the base-7 New Date is the best choice. However, years and months do not need to be expressed in base-7, as they are not concerned with weeks (i.e., the cycle of seven days). Based on the above series of significant advantages of the Neo-Gregorian Calendar — especially the base-7 Neo-Gregorian Calendar — it is imperative that the Neo-Gregorian Calendar replace the current Gregorian calendar. This is particularly true among educated groups, for instance, in academic calendars.

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